Analysis of Coriolis flowmeters effected by cryogenic fluid based on stiffness model

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Abstract

Coriolis Mass Flowmeter (CMF) is investigated in this paper. Based on stiffness model, which is a simply supported beam simplified by the Euler-Bernoulli beam, the mass flow rate expression contained dimensionless function can be deduced. Then, due to the relationship of stiffness parameters and material temperature, the theoretical calculation expression of Flowmeter relative error is reached. The calculation expression is shown by material linear expansive coefficient and temperature difference. Finally, the Flowmeter relative error with the temperature range from *-197 ℃* to *+7 ℃* is calculated.

1. Introduction

With an ever elevating requirement on the precision of measuring liquid natural gas (LNG), Coriolis mass flowmeter, an instrument that can directly measure, is more and more welcomed in LNG measuring and trade for its excellent performance. It can be seen as one of the best meters measuring LNG. Earlier studies on Coriolis mass flowmeter didn't take temperature's effect on the measurement into account. Researchers, manufacturers and users started to alternate their perspective gradually when the range of measurement was extending and the precision was being requiring more often. In 2003, Patten[1] described an operational approach of flow meter at a cryogenic temperature, with the essence lying on rectification of the linear variation of temperature against Young's modulus; in 2008, Kenbar etal[2] reported his result of LNG measurement using Coriolis mass flow meter, in which it is reported that when Young's modulus has a non-linear compensation in the circumstance of low temperature, the measurement error is very close to that resulted from applying a normal temperature; in 2009, Tao Wang and Yousif Hussain[3] drew a conclusion with their experiment that non-linear Young's modulus and thermal expansion of the material of measuring tube will affect the measurement precision; and in 2012, Miao Li and Kejun Xu[4] account in their co-authored article that fluid temperature variation will influence variation of the elastic modulus of the testing tube and further influence the instrument coefficient of the flow meter and the measurement precision. Few studies have been conducted in China on the quantified analysis on Coriolis mass flowmeter with cryogenic fluid. And researchers in international circles also mainly focus on the influence of temperature on Young's modulus of the testing tube. Based on the stiffness model for Coriolis mass flowmeter, the article makes a quantified analysis on Coriolis flowmeter precision affected by cryogenic fluid by studies on the testing tube's Young's modulus and linear expansive coefficient. After the analysis results in mass flow formula including dimensionless function, it also compares with that of Tao Wang and Yousif Hussain's experiment[3].

2. Stiffness Model of Linear Tube CMF

Coriolis forces towards two opposite directions will be generated when the fluid flows through the vibrating straight tube, with the vibration generator as the center. While the Coriolis force at the CMF inlet is opposite against the vibrating direction, making the vibration weakened and delayed, that at the CMF outlet is in concert with the vibrating direction, making the vibration strengthened and advanced. This makes a phase gap of the vibration output at the inlet and outlet. Theoretically, the measured phase gap is proportional to the mass flow. The structural schematic diagram of the Coriolis flowmeter is shown in Figure 1.

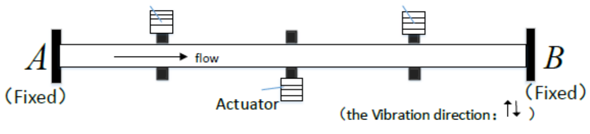

Figure 1: Structure Figure of the Linear Tube CMF .

The measuring pipe is analyzed by simplifying the testing tube into a simple beam model with its two ends fixed and the restraint force is released. It is presumed that the drive exciting force is expressed in the equation $F = F_0 \sin(wt + \theta)$, it can be derived that the displacement equation for the deflection is as follows[5][6]:

$$Y_1(x) = (\frac{F_0 L}{16EI}x^2 - \frac{F_0}{12EI}x^3)\sin(wt+\theta)$$
$$= A\sin(wt+\theta) \qquad 0 \le x \le \frac{L}{2} \qquad (1)$$

In the equation above, E refers to Young's modulus of the straight tube, I to the pipe's moment of inertia, $I = \frac{\pi(D^4 - d^4)}{64}$ [3]; D to the straight tube outer diameter; and d to the inner diameter.

The displacement equation for the measuring tube will be the following when fluid flows through the testing tube and stressed by Coriolis force.

$$Y_2(x) = \frac{F_0}{24E^2I^2}Q_m w(\frac{L}{20}x^5 - \frac{1}{30}x^6 - \frac{L^3}{60})\cos(wt+\theta)$$
$$= B\cos(wt+\theta) \qquad 0 \le x \le \frac{L}{2} \qquad (2)$$

Based on the measuring principle of CMF, the displacement from the exciting disturbance Y_1 and the displacement generated by Coriolis force Y_2 are now overlapped to result in the displacement tested at the two ends as follows:

$$Y(x_0) = A\sin(wt+\theta) + B\cos(wt+\theta)$$
$$= \sqrt{A^2+B^2}\sin(wt+\theta+\beta) \qquad (3)$$
$$Y(L-x_0) = A\sin(wt+\theta) - B\cos(wt+\theta)$$
$$= \sqrt{A^2+B^2}\sin(wt+\theta-\beta) \qquad (4)$$

In the equation above, $0 \le x_0 \le \frac{L}{2}$, β refers to the superposition angle between Y_1 and Y_2; $\tan\beta = w\Delta t/2$, Δt is the time delay between the flow-generated sensor signals at the ends of the testing tube. And,

$$\tan\beta = \frac{B}{A} = \frac{2Q_m w}{EI}(\frac{\frac{L}{20}x^5 - \frac{1}{30}x^6 - \frac{L^3}{60}x^3}{3Lx^2 - 4x^3})\Big|_{x=x_0} \qquad (5)$$
$$= \frac{2Q_m w}{EI}f(\frac{x_0}{L})L^3$$

In the equation,

$$f(\frac{x_0}{L}) = \frac{\frac{L}{20}(\frac{x_0}{L})^5 - \frac{1}{30}(\frac{x_0}{L})^6 - \frac{1}{60}(\frac{x_0}{L})^3}{3(\frac{x_0}{L})^2 - 4(\frac{x_0}{L})^3}$$

When $\frac{x_0}{L}$ is known, $f(\frac{x_0}{L})$ is dimensionless function, invulnerable to temperature. When β is a small value, Equation (5) can be transformed to:

$$Q_m = \frac{EI}{4f(\frac{x_0}{L})L^3}\Delta t \qquad (6)$$

As the equation above expresses the two parameters, the flow meter scale, and Young's modulus, the following paragraph will discuss them affected by cryogenic temperature and further their effect on Q_m. It is presumed that the straight tube linear expansion coefficient is a, the equation expressing the testing tube's Young's modulus varying based on temperature can be approximately represented as follows[7]:

$$E = E_{20}(1 - 25a\delta_T) \qquad (7)$$

In the equation, E_{20} means Young's modulus of the straight tube at the temperature of *20 ℃*, i.e., $\delta_T = T - 20$ ℃. To applying linear expansion coefficient in the calculation of the temperature variation equation for Young's modulus is that linear expansive coefficient can be obtained more easily. The relation between the linear express as $L = L_{20}(1 + a\delta_T)$, $D = D_{20}(1 + a\delta_T)$,and $d = d_{20}(1 + a\delta_T)$. The following equation can be resulted by integrating Equation (7), *I, L,* and *D* into Equation (6).

$$Q_m = \frac{E_{20}I_{20}}{4f(\frac{x_0}{L})L_{20}^3}\Delta t \times (1 + a\delta_T)(1 - 25a\delta_T)$$
$$= Q_{m20}(1 + a\delta_T)(1 - 25a\delta_T) \qquad (8)$$

In the above equation, I_{20} is the sectional moment of inertia of the straight tube. Presuming $E_{20}I_{20} \Big/ 4f(\frac{x_0}{L})L_{20}^3 = C$,when material and structure of the flow meter are determined, C is a constant, with a referable temperature of *20 ℃*.

3. Analysis of Linear Tube CMF Measurement Effected by Cryogenic Fluid

Equation (8) includes two factors, the linear expansion coefficient affected by cryogenic temperature, and Young's modulus, E. When CMF is applied to measure cryogenic fluid like LNG, an error will occur due to the fact that low temperature changes the material linear expansion coefficient "*a*" and Young' modulus "*E*".

$$e = (\frac{Q_m}{Q_{m20}} - 1) \times 100\% \qquad (9)$$
$$= -24a\delta_T \times 100\%$$

It is seen in the above equation, when CMF measures cryogenic fluid, i.e., when $\delta_T < 0$ and $e>0$, the flow meter will display a positive error; contrastively, when $\delta_T>20$ and $e<0$, it displays a negative error. There is a linear proportional relation between the relative error of the flow meter and the temperature difference δ_T, i.e., a larger δ_T brings about a larger error in measurement.

316 stainless steel is taken as an example. Presuming E_{20} =195GPa, the relative error in measurement at a temperature between *-193 ℃* and *7 ℃* is calculated based

on the material linear expansion coefficient “a”. Table 1 below lists the errors.

Table 1: Relative Errors between Linear Expansion Coefficient of Straight Tube (316 stainless steel) and CMF

Temperature	**Thermal Expansion”a” （1/℃）**	**Relative Error （%）**
-193	1.30E-05	6.65
-183	1.33E-05	6.48
-173	1.35E-05	6.25
-163	1.38E-05	6.06
-153	1.39E-05	5.77
-143	1.41E-05	5.52
-133	1.43E-05	5.25
-123	1.44E-05	4.94
-113	1.46E-05	4.66
-103	1.47E-05	4.34
-93	1.48E-05	4.01
-83	1.49E-05	3.68
-73	1.50E-05	3.35
-63	1.50E-05	2.99
-53	1.51E-05	2.65
-43	1.51E-05	2.28
-33	1.52E-05	1.93
-23	1.52E-05	1.57
-13	1.53E-05	1.21
-3	1.53E-05	0.84
7	1.53E-05	0.48

Comparison between the error concluded in Equation (9) and the result of the experiment in Reference [3] is shown in Figure 2.

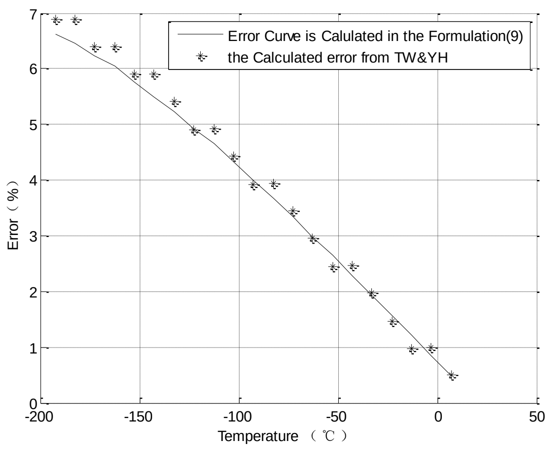

Figure 2 : Measurement Errors of CMF Calculated with Temperature Variation.

It is seen in Figure 2 that the error is maximized, at approximately *0.4%*, when temperature drops to *-183 ℃*.

4. Conclusion

Considering the effect of temperature on Young’s modulus and material linear expansion coefficient, the authors applies dimensionless form to f(x). Usually, Coriolis mass flow meter is rectified at regular temperature with the media of water. However, the measurement precision will be reduced if the material Young’s modulus and linear expansion coefficient affected by cryogenic temperature is not considered. For instance, the theoretical error will be as high as over *6.0%* when measuring LNG at a temperature of *-162℃*. Although the article merely discusses CMF of straight tube, U-shape tube, however, can be seen as one composed of a number of micro straight tubes. The authors therefore believe that the studies on straight tube are referable to the studies of U-shaped CMF.

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