

Dead-Volume Correction (DVC) for the Calibration of CFVNs -- Differential Calibration and Optimum Estimation --

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Abstract

The paper introduces 'differential calibration' that estimates the dead volume correction (DVC) of a diverter valve without ambiguity (termed 'DVC0/DVC1'). DVC0 and DVC1 have to fix the condition of the exhaust line before suction. It is then replaced by a regression method on the basis derived by DVC0 to reduce the number of the measuring points required for the estimation ('DVC1b'). The paper also proposes another regression method that allows any exhausting condition ('DVC2'). It bases on the fact that the discharge coefficient of a CFVN depends on the inverse of the square root of the Reynolds number in the laminar boundary layer regime. Another direct measurement method ('DVC3') is also proposed that can be applied to any conditions. These methods agree with each other very well even between two diverter valves of different sizes.

1. Introduction

Flow is defined by kg/s so that most of the flow standard facilities consist of a mass and a time measurement systems. However, neither of them has any essential relation to the flow through the CFVN during the calibration; the mass measurement is performed on a gas kept in a non-flowing, static condition and the time is just a time interval of two mechanical/electrical/optical events. The reason why such a 'non-flowing/static' facility can measure a quantity of a 'flowing/dynamic' phenomenon is because a flow diverter has converted the dynamic phenomenon into a static one. Being the most difficult thus the last uncertainty to be reduced remained in it, the diverter becomes the most important instrument in a highly-sophisticated facility. All of the dynamic/flow phenomena during a calibration are solely concentrated in the diverter actions.

Any diverter system has a dead-volume (DV) downstream of the CFVN, in which a portion of flowing gas that should not be counted as well as that should be counted are both trapped. The net mass of the gas flowed through a CFVN during a suction time can be obtained only when their corrections are applied on the mass measured by the static tank.

Direct measurement of the mass of gas trapped in a DV during a diverter action is unrealistic because the condition in the DV is highly unstable thus its pressure and temperature vary very rapidly and are distributed.

However, the masses of the gas trapped at the beginning and at the ending moments of a suction time in the DV have no correlation with each other that makes it possible to estimate experimentally the amount of the

dead-volume correction (DVC) by mean of the 'differential calibration', which the paper poposes. As the name implies, the method uses two calibration results to have a single DVC value at a single calibration condition so that it takes a quite long time to cover a certain calibration range. In order to estimate the DVC in a shorter time, a consequence of the differential calibration is then utilized to analyse a simpler sequence of measurements to reach the same DVC by using a regression analysis. Unfortunately, a versatile DVC equation is still not obtained, but each calibration can be corrected very well if the DVC measurement has been performed in advance on a CFVN of the same size.

The paper introduces the following DVC methods;

DVC0 (Measurement): The differential calibration.

DVC1 (Regression): Correction by simply using a fitted line obtained by DVC0.

DVC1b (Regression): Regression method to replace DVC0.

DVC2 (Regression): Regression method to extend DVC1b to any exhaust (EXH) line condition.

DVC3 (Measurements): Extension of DVC0 to any downstream condition.

DVC0 was performed only by keeping the EXH line opened to the atmosphere, thus the application of its resultants is limited only at the same EXH condition. Of course, DVC0 can be performed at any EXH condition, but it will take an unrealistic time to cover whole calibration range. Since DVC1 and DVC1b are based on the DVC0, they can be applied only to the same EXH condition. DVC2 extends to any exhaust conditions by means of a regression analysis by assuming the Reynolds number dependence of the CFVN. The measurement DVC3 bases on the same principle as DVC0 but it estimate the effect of the EXH pressure. DVC3 is expected to result in a versatile DVC equation

that can be applied to any condition, however, it is still under progress.

2. The high-speed diverter valves (HSDVs)

The middle-range air flow standard system in Japan [1] uses a constant volume tank (CVT) system that uses exclusively two high-speed diverter valves (HSDVs) shown in Figure 1. Each HSDV has two poppet valves to divert the flow behind the CFVN. They have the same structure but HSDV2 is larger than HSDV1 as shown in Table 1.

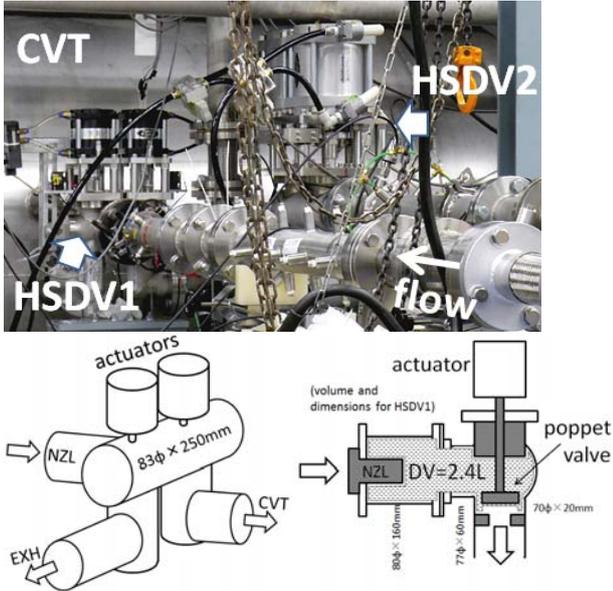


Figure 1: The HSDVs.

Table 1: Specifications of the HSDVs.

	HSDV1	HSDV2
Volume of dead volume	ca. 2.4 L	ca. 5.6 L
Stroking length	15 mm	15 mm
Stroking time	ca. 20 ~ 25 ms	ca. 35 ~ 40 ms
Diameter of flowing area	30 mm	50 mm

3. Calibrations without DVC

The basic principle of the calibration of a CFVN in a CVT system is that the mass of the gas charged into CVT is measured by itself, M_{CVT} , and then divided by the suction time, t_c , resulting in the actual flow of the CFVN, finally it is divided by the theoretical flow of the CFVN, q_{theo} , to obtain C_d . The raw discharge coefficient C_d^{RAW} that exactly follows the principle is given by

$$C_d^{RAW} = \frac{M_{CVT}}{q_{theo} t_c} \quad (1)$$

Examples of C_d^{RAW} of a CFVN when using HSDV1 are shown in Figure 2. The EXH line was always opened to the atmosphere. Being calibrated many times at severe conditions such that $t_c = 15$ s, C_d^{RAW} vary over 0.3%. As seen in the lower left figure, the deviation has some correlation with the final CVT pressure, p_2 . If t_c is long enough, C_d^{RAW} has a good enough accuracy as seen in the lower right figure. 'aCURVE' and 'nCURVE' are

the C_d curves defined in ISO 9300 [2] for the accurately and normally machined CFVNs, respectively. 'sCURVE' is defined in the reference [1].

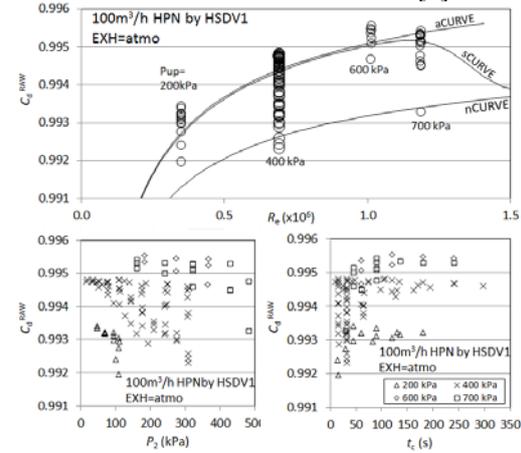


Figure 2: Examples of C_d^{RAW} of a 100 m³/h HPN. HSDV1.

4. DVC (Dead-volume correction)

DVC is directly correlated with the definition of t_c . In the paper, t_c is defined by the time interval from the moment when the EXH side is closed to the moment when the CVT side is closed.

At any moment, DV is trapping a portion of gas. The trapped gas at the starting moment of a suction will be sucked into CVT, but it flowed through the CFVN before the suction time opened, therefore, its mass, M_{DV1} , should be subtracted from M_{CVT} . On the other hand, the trapped gas at the ending moment of a suction did flow through the CFVN in the suction time but will not be sucked into the CVT, therefore, its mass, M_{DV2} , should be added to M_{CVT} . Consequently, the net mass of the gas flowed through the CFVN exactly in the suction time, M_{DVC} , is given by

$$M_{DVC} = M_{CVT} + \Delta M_{DV21} \quad (2)$$

where ΔM_{DV21} is the DVC and given by

$$\Delta M_{DV21} = M_{DV2} - M_{DV1} \quad (3)$$

Since the flow in a DV is extremely disturbed at a moment when HSDV is motivated, it is very difficult to estimate M_{DV1} and M_{DV2} correctly.

5. DVC0 (Differential calibration)

DVC0 estimates DVC experimentally without any hypothesis and also without using any specification of the DV, e.g., its size. In practice, it is supposed that the calibrations are performed in a air-conditioned room thus the temperature effects are negligible. Of course, it is also supposed that the calibrating gas is always the same. It bases on the following apparent facts.

- 1) If the upstream (UP) pressure, p_{up} , of a CFVN is kept the same, its C_d is constant even when the other conditions are different.

- 2) M_{DV1} depends only on the initial EXH pressure, p_{dw} , as long as the CFVN and the structure of the EXH line are kept the same.
- 3) M_{DV2} depends only on the final CVT pressure, p_2 , as long as the CFVN is kept the same.

The facts 2) and 3) are obvious if the paths of the HSDV have no overlap during its action. Of course, the situations affected by the critical back pressure ratio, premature unchoking phenomenon, and so on that changes C_d are carefully excluded. Be sure that p_{dw} is not the pressure in the DV at the starting moment of the suction time but the pressure of the idling flow at some reference location in the EXH line before the suction starts thus p_{dw} is measured when it is constant.

DVC0 uses two calibration results (denoted Calibration A and B) whose p_{up} , p_{dw} and p_2 are identical but their initial CVT pressures, p_1 , are different. In the two measurements on the same CFVN, M_{DV1} are identical because p_{dw} are identical. So as M_{DV2} , because p_2 are identical. q_{theo} and C_d^{DVC} are also identical because p_{up} are identical where C_d^{DVC} is the corrected thus the true C_d . Accordingly, the following equations are made.

$$\text{Calib. A: } C_{dA}^{DVC} = \frac{M_{CVTA} + \Delta M_{DV21}}{q_{theo} t_{cA}} \quad (4)$$

$$\text{Calib. B: } C_{dB}^{DVC} = \frac{M_{CVTB} + \Delta M_{DV21}}{q_{theo} t_{cB}} \quad (5)$$

$$C_{dA}^{DVC} = C_{dB}^{DVC} \quad (6)$$

Solving these equations results in ΔM_{DV21} . Quantities obtained by DVC0 are denoted by a superscript DVC0.

$$\Delta M_{DV21}^{DVC0} = \frac{M_{CVTB} t_{cA} - M_{CVTA} t_{cB}}{t_{cB} - t_{cA}} \quad (7)$$

6. DVC0p (Practical differential calibration)

In practice, p_{up} can be exactly controlled at a desired pressure, but the UP temperature varies slightly depending on the ambient temperature. Since DVC0 bases on a differential of two calibrations that magnifies error, the higher accuracy is critical. To correct the slight difference of the UP conditions, Equation (7) is modified by introducing the Reynolds number dependence of the CFVN under calibration. For a common CFVN, a and b are given in references [1], [2] and so on and they are not necessary to be accurate. Especially a is vanished in the differential calibration.

$$C_{dA}^{DVC} = a - \frac{b}{\sqrt{R_{eA}}} \quad (8)$$

$$C_{dB}^{DVC} = a - \frac{b}{\sqrt{R_{eB}}} = C_{dA}^{DVC} + b \left(\frac{1}{\sqrt{R_{eA}}} - \frac{1}{\sqrt{R_{eB}}} \right) \quad (9)$$

These equations result in the DVC0p equations.

$$\Delta M_{DV21}^{DVC0p} = \frac{q_{theoB} t_{cB} M_{CVTA} - M_{CVTB} q_{theoA} t_{cA}}{q_{theoA} t_{cA} - q_{theoB} t_{cB}} \quad (10)$$

$$+ b \frac{q_{theoA} q_{theoB} t_{cB} t_{cA}}{q_{theoA} t_{cA} - q_{theoB} t_{cB}} \left(\frac{1}{\sqrt{R_{eA}}} - \frac{1}{\sqrt{R_{eB}}} \right)$$

$$C_d^{DVC0p} = \frac{M_{CVT} + \Delta M_{DV21}^{DVC0p}}{q_{theo} t} \quad (11)$$

7. DVC0p applied on a 100 m³/h CFVN calibrations using HSDV1

DVC0p was applied to the calibration results shown in Figure 2 whose conditions are shown in Figure 3. Combinations of two calibration results whose p_{up} and p_2 are identical but p_1 are different enable to perform DVC0/DVC0p. Their C_d^{DVC0p} calculated by using Eq. (11) are shown in Figure 4. Their scattering is small enough even when all the results are included. Removing the results with $t_c < 75$ s, the scattering becomes very small as shown in Figure 5.

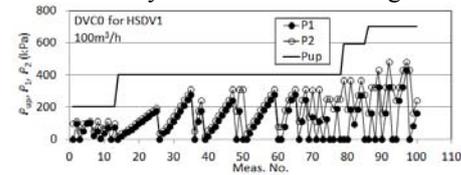


Figure 3: Conditions of the calibrations shown in Figure 2.

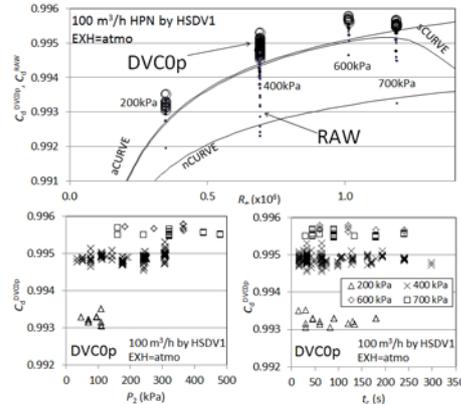


Figure 4: C_d^{DVC0p} of the calibrations shown in Figure 2.

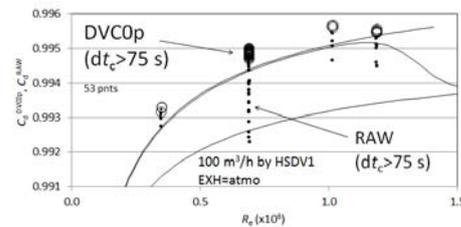


Figure 5: C_d^{DVC0p} of the calibrations shown in Figure 2. $t_c > 75$ s.

8. DVC1

In the DVC0p results, it is shown that ΔM_{DV21} is represented by a line against p_2 as shown in Figure 6. It does not depend on the nozzle flowrate or on p_{up} .

$$\Delta M_{DV21}^{HSDV1/atmo-DVC1} (g) = 0.0374 p_2 (kPa) - 2.24 \quad (12)$$

It is emphasized again that the EXH line was always opened to the atmosphere, therefore, if the EXH line is in another condition, for example, by vacuumed or contracted, ΔM_{DV21} will be represented by another line.

DVC1 uses Eq. (12) to do DVC. An example of the DVC1 results are shown in Figure 7. It is interesting that DVC1 is accompanied by smaller scatterings than DVC0p even though it bases on DVC0p results. This is because DVC1 was filtered by the statistical procedure.

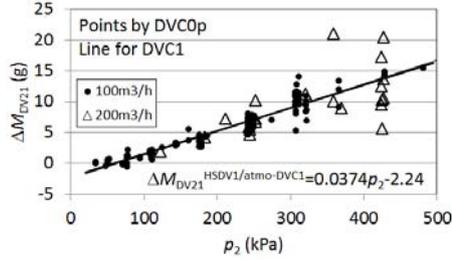


Figure 6: The line representing ΔM_{DV21} obtained by DVC0p.

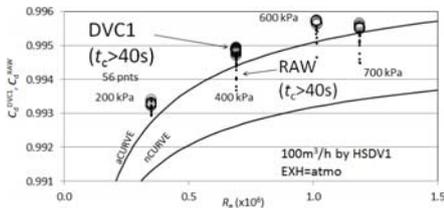


Figure 7: $C_d^{HSDV1/atmo-DVC1}$ of the calibrations shown in Figure 2.

9. DVC1b

Since performing DVC0 takes quite a long time and high concentrations to control the conditions exactly, a statistical method was developed to replace it by utilizing the fact that ΔM_{DV21} is expressed by a line that has a single parameter of p_2 .

$$\Delta M_{DV21}^{DVC1b} = cp_2 + d \quad (13)$$

Since this line does not depend on the CFVN size or p_{up} , it can be measured by a single CFVN of any size at any p_{up} , thus by a single calibration sequence. By

keeping p_{up} constant, C_d^{DVC1b} corrected by ΔM_{DV21}^{DVC1b} is given by

$$C_d^{DVC1b} = \frac{M_{CVT} + cp_2 + d}{Q_{theo} t_c} = const. \quad (14)$$

that becomes

$$cp_2 + d = C_d^{DVC1b} Q_{theo} t_c - M_{CVT}. \quad (15)$$

Using M_{CVT} , Q_{theo} , t_c and p_2 obtained in a sequence of calibrations performed at a constant p_{up} , the best fitted line was searched by scanning C_d^{DVC1b} over an expected range. At the same time when the best fit is found, the parameter c and d as well as C_d^{DVC1b} are already determined. Figure 8 shows the residual errors of the fitting (left) and the best parameters (right) in a calibration sequence on a 100 m³/h HPN by using HSDV2. Their calibration conditions are shown in the center. It was also confirmed that calibrations of less

numbers can result in the same line at a good accuracy. The DVC1b equation for HSDV2 is given by

$$\Delta M_{DV21}^{HSDV2/atmo-DVC1b} (g) = 0.0505 p_2 (kPa) - 6.61. \quad (16)$$

Eq. (16) corrected the calibration results performed by using HSDV2 very well as shown in Figure 9.

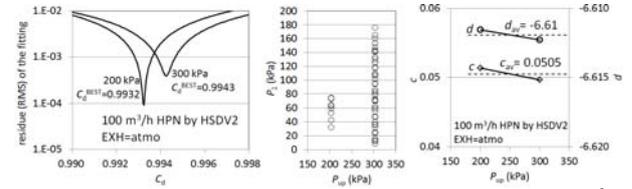
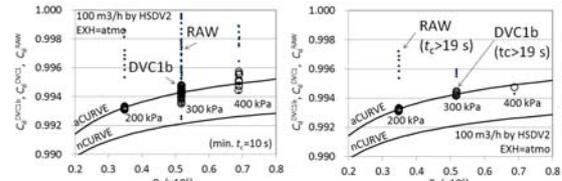


Figure 8: Residual errors and best parameters for DVC1b. 100 m³/h HPN on HSDV2.



(a) All the results. (b) $t_c > 19s$ only.

Figure 9: DVC1b of the calibrations of a 100 m³/h HPN by HSDV2 by using the parameter c and d shown in Figure 8.

10. DVC2

DVC0 and DVC1 are limited to a specified EXH condition thus have a single parameter p_2 . In order to introduce a parameter representing the EXH condition, p_{dw} , which is the downstream pressure during the idling flow before the suction starts, into DVC, DVC2 was developed that uses the similar statistical manner as DVC1b. DVC2 uses calibration results of a CFVN at various p_{up} and p_{dw} . It supposes that the Reynolds dependence of the CFVN is in the form of Eq. (8) and that the dependence of M_{DV1} on p_{dw} is also linear as in Eq. (13). On these assumptions, the next equation is made.

$$M_{CVT} - \left(a - \frac{b}{\sqrt{R_c}} \right) Q_{theo} t_c + cp_2 = ep_{dw} + d \quad (17)$$

As in DVC1b, a , b and c were scanned in each expected range to find a best fit line against p_{dw} . Figure 10 shows the residual errors of the best fit search applied to a calibration sequence of 25 m³/h HPN by using HSDV2. Each residual error has a clear minimum, so that the best parameters are clearly defined.

$$\Delta M_{DV21}^{HSDV2-DVC2} (g) = 0.060 p_2 (kPa) - 0.054 p_{DVO} (kPa) - 0.3 \quad (18)$$

The DVC2 results are compared with DVC1 results applied on calibrations of the same HPN by using HSDV1 are shown in Figure 11. In these calibrations, HSDVs and calibration ranges were different, but they agree quite well with each other. The best fit produced the following DVC1b equation for HSDV2. The best fit search also resulted in C_d^{DVC2} at the same time.

$$C_{dA}^{DVC} = 0.9998 - \frac{3.41}{\sqrt{R_{eA}}} \quad (19)$$

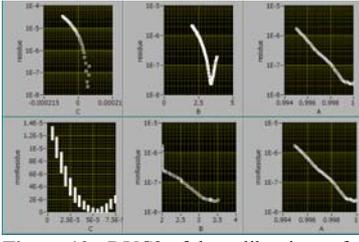


Figure 10: DVC2 of the calibrations of a 50 m³/h HPN by HSDV2.

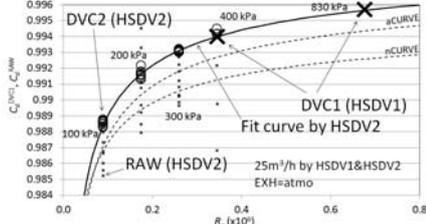


Figure 11: Comparison of C_d^{DVC} of 25 m³/h HPN by DVC2 using HSDV2 and DVC1 using HSDV1.

11. DVC3

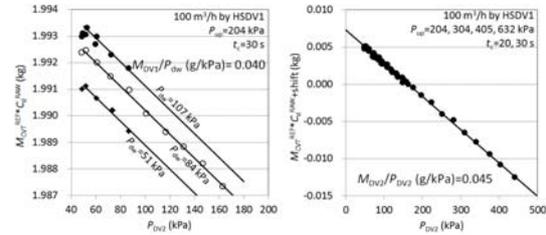
Another DVC measurement, DVC3, are being developed that will be more effective than DVC0 to perform. Because of the effectiveness, it will be able to perform over the whole range of the calibration conditions to derive a versatile DVC equation for a facility. DVC2 can do the same, but it bases on many hypotheses, therefore, it is better confirmed by a measurement.

In DVC3, the parameter p_2 used in DVC1~2 is replaced by p_{DV2} , which is the pressure in the DV at the moment of the suction end. Since p_{DV2} is varying along the time in a charging time, a less accuracy is expected, but the varying rate is not so high and also a high accuracy is not required as well as p_{dw} is used just for an index of the EXH condition, it is found that p_{DV2} is a better parameter to describe ΔM_{DV21} . DVC1~2 will be also recalculated to use p_{DV2} soon. Since the temperature thus the pressure in a CVT gets very high during a charge of gas, p_{DV2} also gets high and finally differs significantly from p_2 that is the pressure when the CVT is cooled down. The difference depends on the suction time and the flowrate that are better to be excluded. Accordingly, p_{DV2} is better than p_2 to describe the DVC.

DVC3 bases on the same facts listed in 5. At constant p_{up} and p_{dw} , variation of C_d^{RAW} of a CFVN is a reflection of solely that of M_{DV2} . Repeating a calibration of a CFVN at various p_{DV2} while keeping p_{up} , p_{dw} and t_c constant, the gradient of C_d^{RAW} against p_{DV2} is a direct reflection of the gradient of M_{DV2} against p_{DV2} . Figure 12 is an example of a result of a sequential calibration, in which, by using HSDV1, calibrations of a 100 m³/h HPN was repeated by

charging into CVT one after another with keeping $p_{up} = 204$ kPa and $t_c = 30$ s, therefore, almost the same M_{CVT} was charged into the CVT at each calibration. p_{dw} was set at 51, 84 or 107 kPa at each sequences. In the results of each calibration sequence, C_d^{RAW} has a linear dependence on p_{DV2} and also an individual shift. By converting to $C_d^{RAW} M_{CVT}^{REF}$ where M_{CVT}^{REF} is a reference M_{CVT} in each calibration sequence (e.g., the averaged M_{CVT}), all the $C_d^{RAW} M_{CVT}^{REF}$ has the same gradient regardless of the calibration conditions. Since M_{CVT}^{ref} varies depending on the calibration condition, each line originally had an individual shift. Cancelling the shift by adding a constant unique to each sequence, all the $(C_d^{RAW} M_{CVT}^{ref} + \text{shift})$ gather along a single line as shown in Figure 12 (b), the gradient of which is $\Delta M_{DV2} / \Delta p_{DV2}$.

$$\frac{\Delta M_{DV2}}{\Delta p_{DV2}} = 0.045 \text{ (g/kPa)} \quad (20)$$



(a) $p_{up} = 204$ kPa.

(b) All the calibrations.

Figure 12: Measurement results for DVC3 on a 100 m³/h HPN by HSDV1.

By using the gradient in Eq. (20), the shift amounts between each line in Figure 12 (a) are calculated. These shift rate against p_{dw} is $\Delta M_{DV1} / \Delta p_{dw}$. In this case,

$$\frac{\Delta M_{DV1}}{\Delta p_{dw}} = 0.040 \text{ (g/kPa)}. \quad (21)$$

It is reasonable to expect that these two gradients must be identical, but letting $\Delta M_{DV1} / \Delta p_{dw}$ at 0.045 or vice versa resulted in apparent deviations of the lines from the measured points in Figure 12 (a), so there must be some fluid dynamical reasons in the difference such as temperature increment in DV caused by the contraction during the HSDV motions. The gradient in Eq. (20) is almost the same as that of DVC1 in Eq. (12) but significantly different indeed. The difference is because p_{dw} in the EXH line that is opened to the atmosphere varies depending on p_{up} when performing DVC0, thus $\Delta M_{DV1} / \Delta p_{DV0}$ and $\Delta M_{DV2} / \Delta p_{DV2}$ were merged into the single gradient in Eq. (12). The same situation is found in the correlation between Eqs. (16) and (20) for HSDV2.

These estimations only give the sensitivities of M_{DV} against p_{DV} or p_{dw} . A constant is expected in the DVC equation but is still not determined. The constant was determined manually by letting the scattering of C_d^{DVC3}

at each p_{up} minimum. Finally, DVC3 equation for HSDV1 estimated by this calibration sequence is given by

$$\Delta M_{DV21}^{HSDV1-DVC3} (g) = 0.045 \Delta p_{DV21} (kPa) - 0.040 \Delta p_{DV1} (kPa) + 1.1 \quad (22)$$

$C_d^{HSDV1-DVC3}$ calculated by using Eq. (2) are shown in Figure 13. Whereas C_d^{RAW} varies over 0.3%, C_d^{DVC3} does only about 0.03% and do not depend on p_{DV2} or p_{dw} .

The C_d^{DVC3} shown in Figure 13 are plotted against the Reynolds number in Figure 14. In this case, the HPN has a boundary layer trip on front of its throat that resulted in an early boundary layer transition thus the C_d is flat along the Reynolds number [3].

The gradients and the constant in Eq. (15) sometimes depend on some individuality of the CFVN. The gradients are mostly OK but another constant than that in Eq. (15) is often better for other CFVNs, especially for small CFVNs. A scientific way to determine the constant and/or the dependence of the DVC3 coefficients on CFVN specifics are under development. Figure 15 is an example where Eq. (15) worked fine. A typical example where another constant fits better is shown in Figure 16.

12. Conclusion

Experimental methods to measure the amount of the dead-volume correction (DVC) for calibrations of CFVN using a charging tank are developed. The method termed 'DVC0' uses differential of two calibration results. It does not use any hypothesis or specification of the DV such as its size, but only a rough pressure in it. DVC0 works very fine but performing it takes a very long time. The other experimental method termed 'DVC3', which is more effective than DVC0 and accept wider range of the calibration condition, was then developed but it is still under development since nozzle geometry seems to affect their correction coefficients by unknown way. However, it works very fine as DVC0 when it is once performed for each CFVN (or for another CFVN of the same size) to determine the individual coefficient set. The authors are searching for the mechanism of the geometry affect to obtain a set of versatile coefficients that can be applied to any calibration condition.

Besides these experimental methods, statistical ones termed 'DVC1', 'DVC1b' and 'DVC2' were also developed based on the resultants of DVC0. DVC1 simply uses the fitted line obtained by DVC0. DVC1b generates the same line by analysing the results of a simpler sequence of calibrations than those needed in DVC0. DVC2 uses the similar analysis as DVC1b but it introduces one more parameters to accept wider range of the calibration conditions, however, it supposes a form of the Reynolds number dependence of the C_d . As far as confirmed, it works very fine.

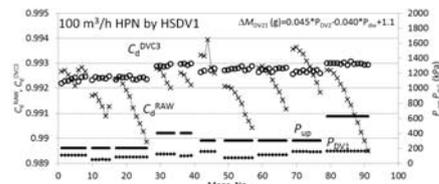


Figure 13: C_d^{DVC3} of a 100 m³/h HPN calibrated by using HSDV1.

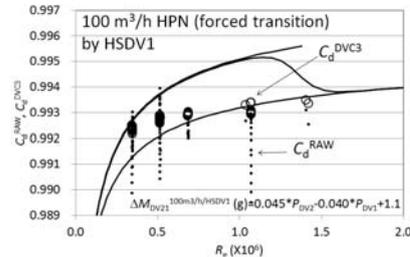


Figure 14: C_d^{DVC3} of a 100 m³/h HPN calibrated by using HSDV1.

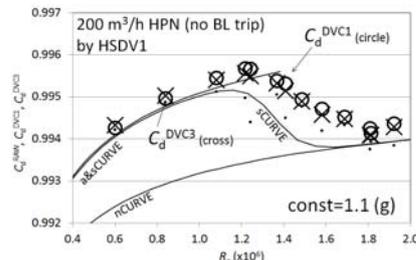
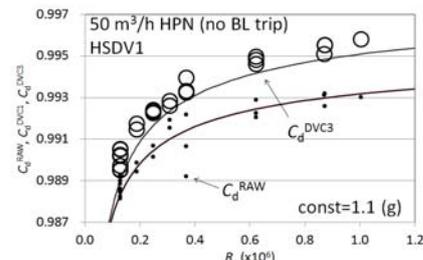
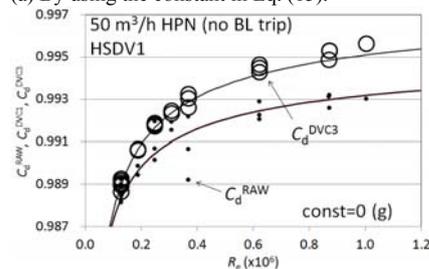


Figure 15: C_d^{DVC3} of a 200 m³/h HPN calibrated by using HSDV1.



(a) By using the constant in Eq. (15).



(b) By using a constant zero.

Figure 16: C_d^{DVC3} of a 200 m³/h HPN calibrated by using HSDV1.

References

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