# Extended data analysis of bilateral comparisons with air and natural gas up to 5 MPa

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**Abstract**

In 2015, the PTB, LNE-LADG, NIM and NIST performed informal bilateral comparisons using six critical venturi nozzles (CFVN). The goal of the comparisons was to prove the equivalence of reference standards for gas flow using pressurized air and natural gas at pressures from 0.1 MPa to 5 MPa. The evaluation of the data utilized a function for the discharge coefficient $c_D$ of critical nozzles as a function of the Reynolds number and real gas characteristics. The fitting function covers both laminar and turbulent boundary layer operating ranges of a nozzle with a single equation. The summary of all results of this comparison series shows that the proposed function for the discharge coefficient $c_D$ can represent single measurement values within 0.1 % with a 95 % confidence level.

The approach represents the results reported by participants using functions based on physical understanding of the transfer standards. The paper documents the full equivalence of all traceabilities involved in these comparisons. The comparison database includes not only measurement sets of 2015 but also measurements done by PTB in other years with the same nozzles. Hence, qualitative statements on the long term reproducibility could inform the analysis on multilateral equivalence of the participants. Linked bilateral comparisons can effectively expose discrepant labs and perhaps give insight on the long term drift of primary standards.

## 1. Introduction

In the past 15 years, the National Metrology Institutes (NMIs) and Designated Institutes (DIs) made substantial efforts to establish a system of intercomparisons to support the quality of calibration results under the Mutual Recognition Arrangement (MRA) organised within the Bureau International des Poids et Mesures (BIPM, www.bipm.org).

Besides high level key comparisons, many NMIs or DIs in the field of fluid flow continue to do informal comparisons, performed bilaterally and efficiently at a less sophisticated technical level. These comparisons can provide a database to make the calibration capabilities evident.

The comparisons discussed herein were organised ad-hoc in 2015 between 1) PTB and LNE-LADG[1], 2) PTB and NIM, and 3) NIM and NIST using a total of six critical nozzles. See Table 3 for definitions of acronyms. A special characteristic of the database generated within this series of measurement is the wide range of pressure (0.1 MPa to 5 MPa) and the different gas species (air and natural gas) used for the calibrations. Furthermore, PTB could provide datasets of calibrations for five of the nozzles performed in previous years and based on consistent traceability to the same primary standards.

Hence, a large database was available and it was the central challenge to make use of all these data to determine the degrees of equivalence between the participants. Therefore, it was necessary to introduce new approaches based on the application of functions to represent the values of $c_D$ reported by the NMIs.

## 2. Overview of the transfer standards and the NMIs

The transfer standards for the comparison measurements were critical nozzles of different throat diameters. Both types of nozzles defined by the ISO 9300 [1], toroidal and cylindrical, have been used. To keep the comparisons simple, less expensive, and low risk regarding damage or loss of equipment, only the nozzles were transported among the labs. Consequently, all auxiliary equipment (pressure, temperature, and composition sensors) necessary for the complete determination of the discharge coefficient $c_D$ were provided by each participating laboratory. Each participant included the uncertainty of the auxiliary instruments in their

[1] LNE-LADG is the Laboratoire National de Métrologie et d'Essai, Laboratoire Associé de Débitmétrie Gazeuse. CESAME EXADEBIT is designated institute of LNE-LADG.

uncertainty of the reported $c_D$ values. Table 1 and Figure 1 give an overview to the six nozzles and the mass flow range covered. From these data, it is apparent that at mass flows of approximately 100 kg/h, there is indirect linkage from NIST through NIM and PTB to LNE-LADG via three or more CFVNs. Our goal is to advance the analysis of this bilateral comparison database.

Table 2 lists the NMIs and their primary standards that are relevant for the comparisons discussed here. All NMIs have already established CMCs in the BIPM database, but CMC revisions are in process:

- NIM has increased the operating pressure to 2.5 MPa [3].
- LNE has completely re-built their *pVTt* system and is using air [6].
- PTB offers direct calibrations of customer devices with their primary standard for high pressure natural gas.

It is one intention of this paper to provide evidence regarding these ongoing changes of the related CMC entries.

**Table 1:** Nozzles used in the comparisons.

| CFVN | *d* [mm] | Form | NIST | NIM | PTB | LNE |
|---|---|---|---|---|---|---|
| NIST 2.5 mm | 2.5 | toroidal | X | X | | |
| HD-17b | 2.156 | toroidal | | X | X | |
| HD-9b | 4.9452 | toroidal | | X | X | |
| HD-5b | 6.9882 | toroidal | | X | X | |
| TF65 | 10.007 | cylindrical | | | X | X |
| TF200 | 17.396 | cylindrical | | | X | X |

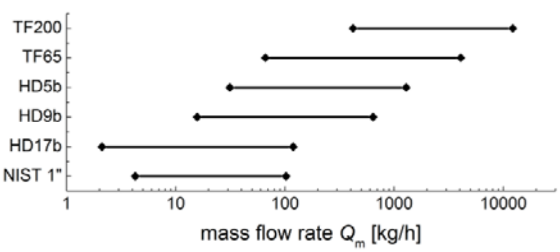


**Figure 1:** Ranges of mass flow covered by the different nozzles within the comparison campaigns.

**Table 2:** List of the NMIs and their primary standards.

| NMI | Gas used | Primary standard | Ref. | Maximum pressure [MPa] |
|---|---|---|---|---|
| NIST | Air | *pVTt* | [2] | 0.7 |
| NIM | Air | *pVTt* | [3] | 2.5 |
| PTB | Air<br>Natural gas | Bell prover*<br>piston prover** | [4]<br>[5] | 0.8<br>5.6 |
| LNE | Air | *pVTt* | [6] | 4 |

* Working standards were used for the calibrations in all measurements with air above 100 kPa.

** Working standards were used for the calibrations in all measurements with natural gas before 2015.

## 3. On the uncertainties in comparisons and the degree of equivalence

The Working Group for Fluid Flow (WGFF) in the Consultative Committee for Mass and Related Quantities at the BIPM, released a guideline [7] for CMC and Calibration Report Uncertainties in 2013. According to the role of the BIPM, the statement about the uncertainty in a CMC entry in the BIPM database has to include the repeatability $u_{\mathrm{repeat,BED}}$ of a Best Existing Device (BED) under test in addition to the base uncertainty of the laboratory's reference standard:

$$u_{CMC}^2 = u_{base}^2 + u_{repeat,BED}^2 . \tag{1}$$

The Transfer Standard (TS) used in an intercomparison will have a different repeatability performance $u_{\mathrm{repeat,TS}}$ than the laboratory's BED. Consequently, the uncertainty reported by a laboratory $u_{\mathrm{Lab,reported}}$ in a comparison is given by:

$$\begin{aligned} u_{Lab,reported}^2 &= u_{CMC}^2 - u_{repeat,BED}^2 + u_{repeat,TS}^2 \\ &= u_{base}^2 + u_{repeat,TS}^2 . \end{aligned} \tag{2}$$

In practice, transfer standards have a certain long term instability $u_{\mathrm{TS,stab}}$ which is not represented by the $u_{\mathrm{repeat,TS}}$ and has to be determined by repeated calibrations covering a long period. Therefore the value $u_{\mathrm{Lab,applied}}$ finally used for the calculation of the degree of equivalence includes this additional contribution to the uncertainty:

$$u_{Lab,applied}^2 = u_{base}^2 + u_{repeat,TS}^2 + u_{TS,stab}^2 . \tag{3}$$

The (bilateral) degree of equivalence is the central outcome of the comparisons and is expressed here as the normalized difference between the values reported by two laboratories:

$$E_{N,Lab1,Lab2} = \frac{c_{D,Lab1} - c_{D,Lab2}}{k\sqrt{u_{Lab1,applied}^2 + u_{Lab2,applied}^2}} , \tag{4}$$

where the combined uncertainty of both participants is the normalization factor using the expansion factor $k = 2$ so that we get the 95 % coverage interval. The critical level for abs($E_N$) is therefore 1. Exceeding the level of 1 indicates the non-equivalence of the calibration results.

In the past, some effort was necessary to make sure that all participants performed their measurements under very similar working conditions. Specifically the operating pressure, flow, and fluid composition or properties had to be similar for all participants because the indication performance of our transfer standards is dependent on these operating conditions. Hence, the comparisons were evaluated in a so-called point-to-point (P2P) manner, meaning that only values were compared which were generated at very similar operating conditions for all participants. The database generated within the series of measurements documented in this paper does not fulfill the requirements for such a P2P evaluation. Therefore, we have chosen to pursue an approach that determines functions that which represent the data of the participating labs.

Least Squares Fit (LSF) functions are practical solutions, but require the consideration of the following topics to avoid misinterpretation of the results:

- Correlation among data measured with common traceability (data out of one lab),[2]
- Bias effects due to imbalances regarding number of data provided by the labs (the results can be biased towards the larger datasets),
- Choice of the fit function. The structure of the fit function shall have sufficient complexity to cover the characteristics of the true relationship (a demonstrated physical model) but shall also provide results based on a sufficient statistical significance.
- Extrapolation to very different conditions of flow and composition increases uncertainty.

Our approach here was to use a function which shall represent the general behavior of the dependency of our measurand, the discharge coefficient $c_D$ of a critical nozzle as a function of the Reynolds number $Re$. This first approximation $c_{D,base} = f(Re)$ is determined by means of a weighted LSF with data from all available labs for each nozzle. The uncertainties $u_{Lab,reported}$ were used for the weights but we did not consider any correlation among the data (LSF with uncorrelated data). As a consequence, the result for $c_{D,base}$ is definitely biased in multiple ways and the resulting uncertainty of the fit result is strongly underestimated. Therefore, a more sophisticated approach is needed before using this function to produce a comparison reference value.

But we can make use of $c_{D,base}$ to remove a common base line from all single measurement values:

$$\Delta c_{D,Lab} = c_{D,meas} - c_{D,base}. \tag{5}$$

In a second step, the remaining single data $\Delta c_{D,Lab}$ are approximated again by means of a LSF:

$$\Delta c_{D,fit,Lab}(Re) = \mathrm{LSF}(\Delta c_{D,Lab,i}, Re_i). \tag{6}$$

The complexity of functions being necessary to represent the $\Delta c_{D,Lab}$ is dramatically reduced when the first estimation $c_{D,base}$ is quite close to the real behavior of $c_D(Re)$. For this database, it was possible to apply polynomials of first order in most cases and higher order only in a few cases (the maximum was third order in one case).

The LSF processing of the data provides finally the parameters for the fitted function and the variance-covariance matrix of these parameters. With this we generated the values of the fitted curve $\Delta c_{D,fit,Lab}(Re)$ and the related uncertainty to this value $u_{conf,fit}(Re)$ for a new set of Reynolds number in the Reynolds range covered by two laboratories. At each "new" Reynolds number we applied then the conventional approach for the P2P determination of the degree of equivalence Equation (9).

We did not perform extrapolation beyond the range of Reynolds number provided by a participating laboratory.

Please note that the outcome of LSF for the uncertainty of the fitted value at confidence level $u_{conf,fit}(Re)$ has similar meaning as the standard uncertainty of the mean which is background for the $u_{repeat,TS}$ reported by the lab. Furthermore, we assume that critical nozzles are very stable artifacts and it is not necessary to include a value for $u_{TS,stab}$ for the short duration of these comparisons. Hence, we assumed the following simplifying relations for all evaluations documented here:

$$u^2_{repeat,TS} \Rightarrow u^2_{conf,fit} \text{ and } u^2_{TS,stab} \Rightarrow 0 \tag{7}$$

$$u^2_{Lab,fitted} = u^2_{Lab,applied} = u^2_{base} + u^2_{conf,fit}. \tag{8}$$

The bilateral degree of equivalence is finally:

$$E_{N,Lab1,Lab2} = \frac{\Delta c_{D,fit,Lab1} - \Delta c_{D,fit,Lab2}}{k\sqrt{u^2_{Lab1,fitted} + u^2_{Lab2,fitted}}}. \tag{9}$$

## 4. Definition of $c_D = f(Re)$ covering wide Reynolds range

The discharge coefficient $c_D$ depends mainly on the Reynolds number of the flow through the nozzle. Looking to the characteristics of this dependency we have to distinguish two main cases, when the boundary layer of the flow inside the nozzle is laminar ($c_{D,lam}$) and when the boundary layer is turbulent ($c_{D,turb}$):

$$c_D = s_a\left(a + b_{lam}\cdot Re^{-0.5}\right) + s_e\left(a + b_{turb}\cdot Re^{-0.139}\right), \tag{10}$$

with
$$s_a = 0.5\left\{1 - \tanh\left[k_u \log\left(Re/Re_{tr}\right)\right]\right\}$$
$$s_e = 0.5\left\{1 + \tanh\left[k_u \log\left(Re/Re_{tr}\right)\right]\right\}.$$

The parameter $a$ represents the impact of the inclination of the isotach lines (lines of equal speed) at the nozzle throat [9] to the discharge coefficient $c_D$ and $b_{lam}$ as well as $b_{turb}$ indicate the dependency of $c_D$ on Reynolds number in the case of laminar or turbulent boundary layers respectively (for discussion regarding the exponent 0.139 for turbulent boundary layers please refer *e.g.* to [10]).

It is common to bridge the laminar and turbulent boundary layer transition with one function for the whole Reynolds range by means of a transition function given by the two terms $s_a$ and $s_e$. Note that $s_a + s_e \equiv 1$ . The parameter $Re_{tr}$ defines the middle point of the transition and $k_u$ the "sharpness" of the transition (the larger $k_u$, the more "sudden" the transition occurs).

[2] Correlation among data can be included in application of LSF in principle [8] if we would have the prior information about the structure and level of correlations. However this approach needs the prior analysis of the uncertainty budgets .

Equation 10 defines our base function for $c_D$ for a particular nozzle versus all Reynolds numbers which we call in the following, $\boldsymbol{c_{D,base}}$. We will use the database from the participating labs to determine the free parameters in Equation (10) for each nozzle. In many cases, the fitted values for some of the free parameters (and particularly $b_{turb}$) are more difficult and less reliable because of a lack of comparison data in the turbulent regime. Also, the sensitivity to the parameter $k_u$ is weak. Therefore, some reduction of the independent parameters is applied:

- the parameter $k_u$ is fixed arbitrarily to the value $k_u = 5.5$ for all nozzles, and
- we introduce a fixed relationship between $b_{lam}$ and $b_{turb}$.

Based on previous theoretical and numerical studies [10], the following fixed relation between the parameters $b_{lam}$ and $b_{turb}$ can be used:

$$b_{turb} = 0.003654 \cdot b_{lam}^{1.736}, \tag{11}$$

removing $b_{turb}$ from the list of parameters to be determined for each nozzle.

The measurements from our bilateral comparisons cover air and natural gas up to a pressure of 5 MPa. Therefore, real gas effects are significant and an inclusion of gas characteristics as a function of composition and pressure is necessary[3]. For the parameter $b_{lam}$ we made use of the theoretical work of Geropp [11]. Geropp showed that the parameter $b_{lam}$ has dependencies on the isentropic exponent $\kappa$, the Prandtl-number $Pr$, the difference of the wall temperature to ideal condition $\Delta T_{Wall}$, and the radius of curvature of the nozzle inlet near the throat $R_{throat}$. Explicit equations for these dependencies are found in reference [11]:

$$b_{lam} = G(\kappa, Pr, \Delta T_{Wall}) \cdot R_{throat}^{0.25}. \tag{12}$$

Up to now, we have no detailed knowledge about $\Delta T_{Wall}$ therefore we are ignoring the influence of heat transfer on $b_{lam}$ and assume always $\Delta T_{Wall} = 0$.

To utilize the relation given by Equation (12) for our application, we use a parameter $b_{lam,0}$ which is assumed to be valid for the specific gas parameters $\kappa = 1.4$ and $Pr = 1$. This yields:

$$\frac{b_{lam}(\kappa, Pr)}{b_{lam,0}} = \frac{G(\kappa, Pr)}{G_{\kappa=1.4, Pr=1}} = C_b(\kappa, Pr). \tag{13}$$

Hence, for a certain value of $b_{lam,0}$ we can apply the related $b_{lam}$ specific for each gas and each pressure when we calculate $c_{D,base}$.

For the parameter $a$, representing the impact of the inclination of the isotach lines, we apply a similar approach. The parameter $a$ also depends on the isentropic exponent $\kappa$ and the radius of curvature at nozzle throat $R_{throat}$:

$$a = H(\kappa, R_{throat}). \tag{14}$$

An explicit equation for this dependency is given *e.g.* by [9].

Analogous to the approach for $b_{lam}$, we use a parameter $a_0$ assumed to be valid for $\kappa = 1.4$ and which is related to the actual parameter for a specific, actual $\kappa$ according to:

$$\frac{a}{a_0} = \frac{H(\kappa, R_{throat})}{H_{\kappa=1.4, R_{throat}}} = C_a(\kappa, R_{throat}), \tag{15}$$

with $$R_{throat} = \left(\frac{b_{lam,0}}{G_{\kappa=1.4, Pr=1}}\right)^4.$$

Hence, the determination of the function $c_{D,base}$ for one nozzle needs now the determination of three parameters $\{a_0, b_{lam,0}, Re_{tr}\}$ instead of the original five.

The characteristics of our definition of $c_{D,base}$ have important consequences for our purpose here. The relationships of Equation (13) and Equation (15) introduce systematic dependency on the fluid parameters isentropic exponent $\kappa$ and the Prandtl number $Pr$. These parameters differ significantly for different gases as well as for different pressures of the same gas (because of real gas effects). An illustration is given for the nozzles investigated here in the Appendix, Figure 7 to Figure 11. The $c_{D,base}$ in these graphs is calculated using the same $a_0$ and $b_{lam,0}$ for both air and natural gas (NG). But at the high Reynolds numbers, i.e. at the higher pressures, the difference for $c_{D,base}$ between air and natural gas is approximately 0.075% which is of the same order as the expanded uncertainty of the single values. It would definitely distort the results for comparisons using air and natural gas if we ignored this behaviour.

## 5. Results for the Degree of Equivalence $E_N$

All data sets of the nozzles have been treated with the principles given above in Sections 3 and 4. The related database from the six nozzles can be seen in graphical form in the Appendix, Figure 7 to Figure 11. The database is available from the authors upon request.

Figure 2 and Figure 3 below illustrate the $\Delta c_D$ values for one of the six nozzles (HD-9b) and its final $E_N$ curves. The $E_N$ curves in Figure 3 and elsewhere in this paper have been determined with data sets that are pair-wise independent regarding their traceability; so *e.g.* NIM (2015 air) with PTB (2015 air) but also with PTB (2015 NG) and with PTB (2006 air). To avoid introducing uncertainty due to extrapolation, the $E_N$ values are calculated only for the range of Reynolds numbers where the data sets overlap.

[3] This should not be confused with the influence of real gas characteristics which is taken into account by the critical flow factor $C^*$, see *e.g.* [12].

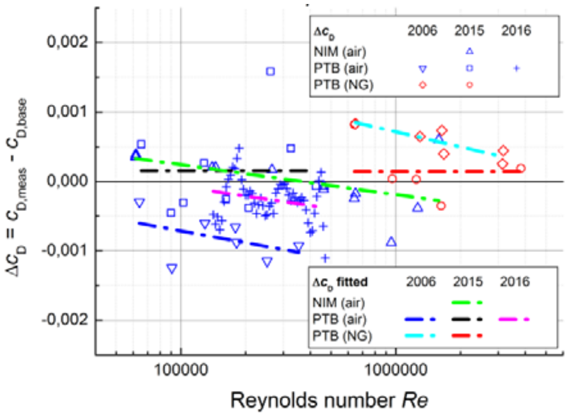


**Figure 2:** Differences $\Delta c_D$ of single $c_D$ values against the $c_{D,base}$ function for the nozzle HD-9b. The $\Delta c_D$ for each measurement series are approximated by a polynomial $\Delta c_{D,fit}$. Please refer to Figure 9 in the Appendix for plot of $c_{D,meas}$ and $c_{D,base}$.

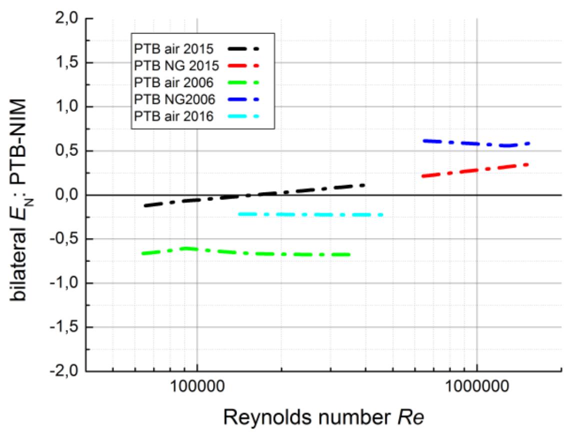


**Figure 3:** Bilateral $E_N$ PTB – NIM using nozzle HD-9b. $E_N$ is calculated according to Equation (9).

The $E_N$ curves in Figure 3 for the bilateral equivalence between PTB and NIM are all significantly within ±1 and vary approximately ±0.5 against an average. This indicates the overall stability of the measurment process including the reference standards of participants and the transfer standard over all the years from 2006 to 2016!

Figure 4 shows the $E_N$ curves for all the the data for all nozzles. To maintain readability, the curves are distiguished by colour based on the bilateral comparison pairs.

All $E_N$ are within ±1 and the pair-wise equivalence between PTB/NIM, PTB/LNE, and NIM/NIST is evident.

After the calculation of the $E_N$ curves and the approval of bilateral equivalence we can ask about the multilateral equivalence. The multilateral equivalence is here evaluated qualitatively[4] using the fact that the bilateral comparisons are establishing a chain between NIST-NIM-PTB-LNE in which NIM and PTB can be used as the link between the other two participants.

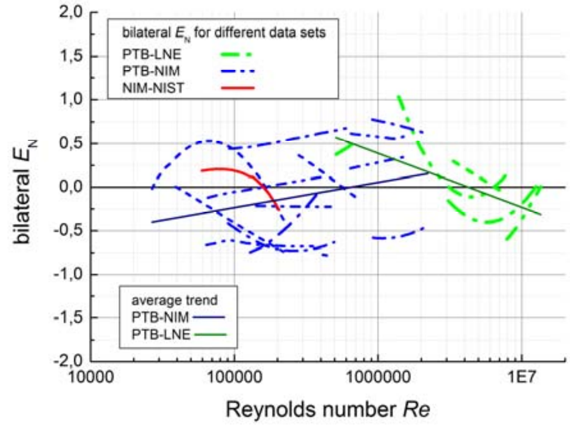


**Figure 4:** Bilateral $E_N$ PTB-LNE, PTB-NIM and NIM-NIST for all nozzles and all datasets.

Looking to Figure 4, we can conclude first that there is only a slight trend in $E_N$ with respect to Reynolds number for all compared data sets between NIM and PTB. Secondly, considering the data PTB-LNE, we can see a trend starting from about +0.6 going down to -0.3. Both trends are getting rather close together in the overlapping range. Hence, we can conclude that also for NIM-LNE we can expect equivalence in the operation range where both NIM and LNE can provide calibrations.

For NIST-NIM we had only one data set and the line of $E_N$ for this case is also reasonably close to the overall trend line for NIM-PTB.

## 6. Evaluation of all single $c_D$ values using $c_{D,base}$

In the following, we will use the overall function $c_{D,base}$ in the sense of a comparison reference value, although formally we are not allowed to name it so as already mentioned above in Section 3. Hence, we will talk about the normalised differences $\Delta c_D/U_{Lab,reported}$ (Figure 6) which have the same philosophy behind them but may slightly differ from the exact values for $E_N$. However we can assume that the difference is small and probably $\Delta c_D/U_{Lab,reported}$ will be overestimated because an existing bias is increasing the values for at least one of the participants.

Figure 5 shows the relative differences $\Delta c_{D,rel}$ against the $c_{D,base}$ functions for all measurements. It can be seen that the majority of the values for $\Delta c_{D,rel}$ are approximately in a range of 0.1%. There are only a few values between 60 kg/h and 200 kg/h that exceed this limit significantly (up to +0.3%[5]).

[4] The trend lines in Figure 4 are given only for orientation because we have not quantified the uncertainties of these average lines.

[5] These values belong to the measurements with nozzle HD-5b at PTB in 2015. This is the only case where we have to assume instability of

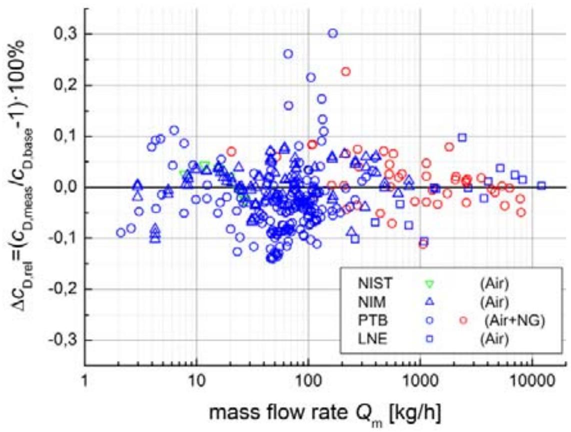


**Figure 5:** Differences $\Delta c_D$ of single $c_D$ values (relative in %) against the $c_{D,base}$ functions for all nozzles and all measurement series.

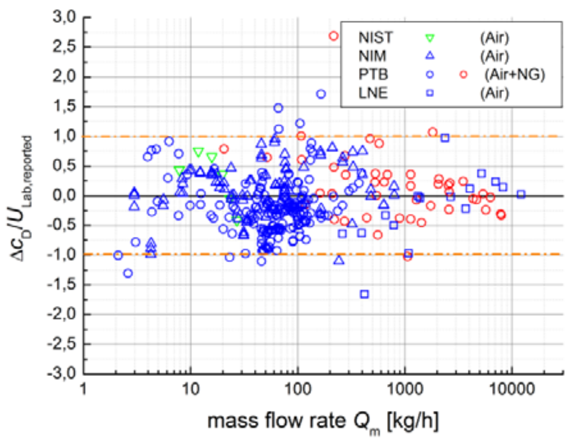


**Figure 6:** Normalised differences $\Delta c_D/U_{Lab,reported}$ as shown in Figure 5 using the related uncertainties reported by the laboratories ($k$ = 2) for all nozzles and all measurement series.

The normalised differences $\Delta c_D/U_{Lab,reported}$ are shown in Figure 6. The overall histogram statistics for these data are shown in the Appendix, Figure 12, and indicate that 95 % of the values are within ±1.[6]

Additionally we can conclude that the functionality behind $c_{D,base}$ is sufficient to represent a single experimental datum at the level of 0.1 % because otherwise the normalised difference is highly unlikely to give abs($E_N$) values in the manner shown here.

## 7. Conclusions

First of all, we conclude from these comparisons that there is equivalence between all the measurement capabilities involved. These results support of the following improved CMCs: 1) LNE with their new *pVTt*-facility [6] with pressure up to 4 MPa and uncertainties of $U_{base} \geq 0.1\%$, 2) NIM with their *pVTt* [3] up to 2.5 MPa and $U_{base}$ = 0.08% and 3) PTB's capability to calibrate sonic nozzles with their High Pressure Piston Prover with $U_{base}$ = 0.072%.

Functions were successfully introduced into the evaluation of the comparison database. This approach was necessary because the database did not allow the application of the conventional approach of point-to-point evaluation. The procedure to first subtract a base line ($c_{D,base}$) from the reported results is very helpful; it reduces the complexity for the representations of data of each laboratory.

It has to be emphasized hereby that the additional contributions of uncertainties caused by the transfer standard to the final values of $E_N$ are reduced because the value for the uncertainty of the fit $u_{conf,fit}$ is replacing the repeatability $u_{repeat,TS}$. In practise, $u_{conf,fit}$ is smaller than the repeatability at one point due to the higher degrees of freedom and is also representing some parts of the $u_{TS,stab}$. Reducing the uncertainty due to the transfer standard is important advantage of this approach for future comparisons because it enhances our ability to assess CMCs based on the comparison results (see for this [13]).

Last but not least, the application of a generalised function to represent the database of discharge coefficients was demonstrated and with the set of overall results it was shown that this function can reflect the experimental data at a precision level of 0.1% with 95 % confidence. Furthermore, the function includes additional real gas effects on the boundary layer that were used to correct significant differences (up to 0.075 %) between application with air and natural gas at high pressures (2 MPa to 5 MPa).

**Table 3.** Definitions of Acronyms.

| | |
|---|---|
| BED | Best Existing Device |
| BIPM | Bureau International des Poids et Mesures |
| CFVN | Critical flow venturi nozzle |
| CMC | Calibration Measurement Capability |
| DI | Designated Institute |
| LNE-LADG | Laboratoire National de Métrologie et d'Essai, Laboratoire Associé de Débitmétrie Gazeuse |
| MRA | Mutual Recognition Arrangement |
| NG | Natural gas |
| NIM | National Institute of Metrology, China |
| NIST | National Institute for Standards and Technology |
| NMI | National Metrology Institute |
| PTB | Physikalisch-Technische Bundesanstalt |
| WGFF | Working Group for Fluid Flow |

the nozzle itself because the values both of the independent measurement series with air and natural gas show these changes and higher scatter than usual. We could not detect the physical reason; during the repeated measurements in 2016 all values were consistent again.

[6] This is in very good consistency with the precondition that the evaluation have been done at a level of 95% confidence for the uncertainties. Also the distribution of the data is similar to the expectation of a random process with a Gaussian distribution.

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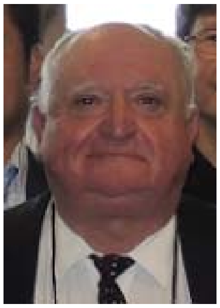
### In Memoriam of Jean-Pierre Vallet



With great sorrow, we received the message that our friend and colleague Jean-Pierre Vallet passed away on July 5, 2016. Jean-Pierre was a kind and generous friend to many of us and his death is a big loss for the flow community.

Critical flow nozzles and their application in flow metering was one of his central topics over all the years, always encouraging cooperative research on his favorite flow meter. Besides advancing technical issues, he also had tremendous abilities to identify strategic needs and to pursue them with long lasting energy.

But in all these long years of his active participation in our community, it was not only his ambition to bring forward our topics in fluid flow that impressed us, but even more, his great joie de vivre, ability to enjoy life itself with a deep sense for real friendship and good lifestyle. With this, we will keep him always in our memory as a great example of a fulfilled life.

## <u>Appendix</u>: Graphical presentation of all measurement results

Results of $c_D$ values determined by the participants and the $c_{D,base}$ function (see Section 4) based on a LSF are given in Figure 7 to Figure 11. We refrain from showing the error bars for the single measurement values to keep the graphs readable.

Figure 12 shows the histogram statistics for data in Figure 6.

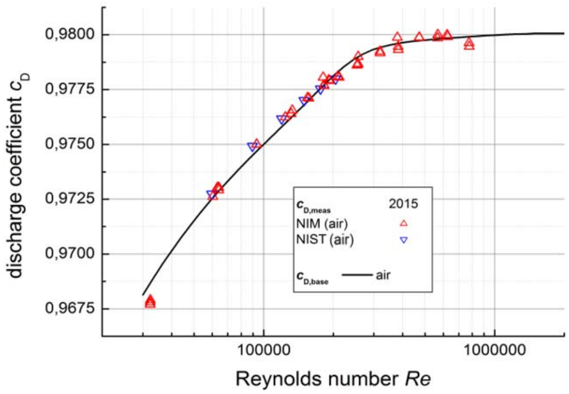



**Figure 7:** Data for nozzle NIST 2.5 mm.

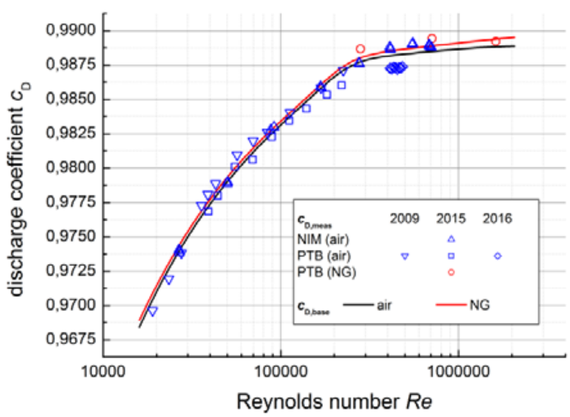



**Figure 8:** Data for nozzle HD-17b.

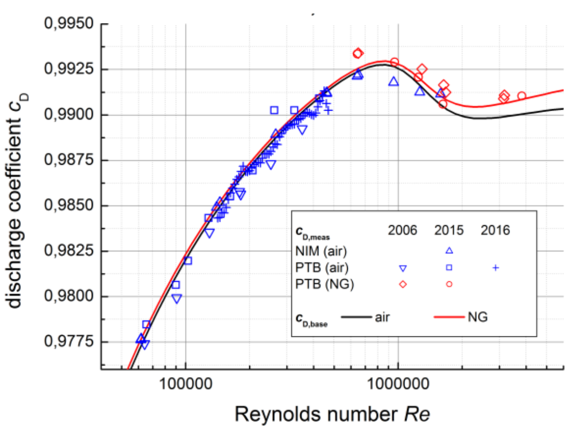



**Figure 9:** Data for nozzle HD-9b.

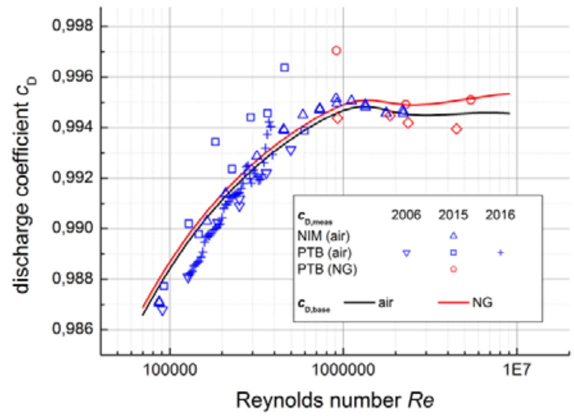



**Figure 10:** Data for nozzle HD-5b.

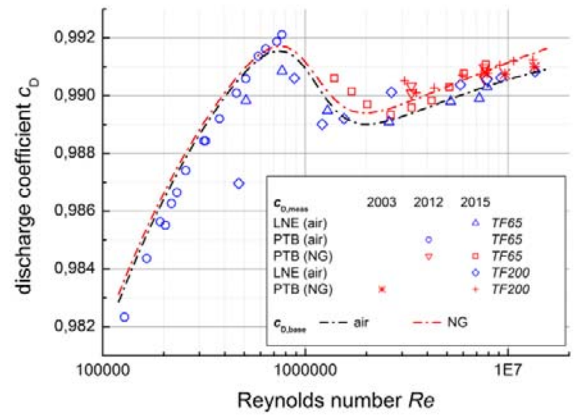



**Figure 11:** Data for nozzles TF65 and TF200. Please note that only one parameter triplet $\{a_0, b_{lam,0}, Re_{tr}\}$ has been applied for both nozzles.

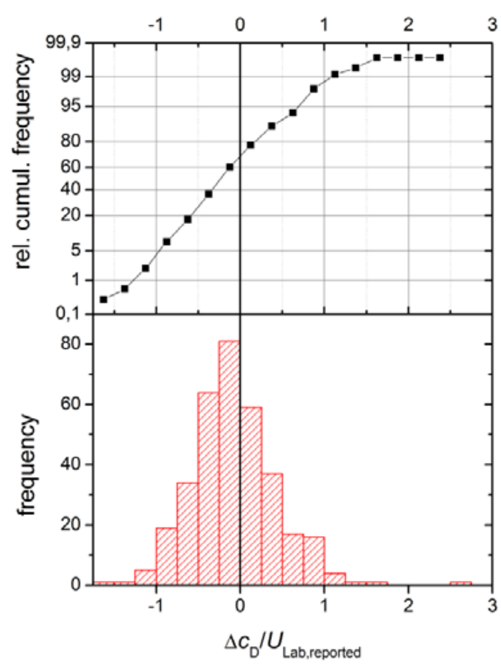



**Figure 12:** Histogram statistics on the data shown in Figure 6.